

Higgsless EW gauge-boson masses from K-K geometry with torsion

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Abstract

In the classical Higgsless $M^4 \times S^3 \times S^1$ Kaluza-Klein gauge-theory vacuum, torsion-induced loss of the $SU(2) \times U(1)$ left-invariance generates a gauge-boson mass term, in co-existence with the gauge-field kinetic term in the vacuum Hilbert-Einstein action. This is compulsory in the sense that having one of these terms without the other would mean violation of basic theorems on holonomy. The effect could be lost only at the very high massless-EW energy scale, where the ground-state manifold approach for the gauge-theory vacuum would be inapplicable. Created, as they are, from and as part of pure geometry, these gauge-boson masses reproduce precisely the spectrum of the well-known experimental result. The apparently exploitable geometric elegance and the ensuing fundamentally geometric origin of mass may provide new theoretical and experimental perspectives around and well above the EW scale.

1 Introduction

In the wake of flow of LHC results, optimism and doubt mix as the Higgs sector of the Glashow-Weinberg-Salam model for the EW interaction [1] undergoes decisive tests and exploration. The main task is, of course, a formidable one, for all the known and spotlighted reasons; if confirmed, it will mark one more triumph in the best tradition of progress in physics. All the same, it is also a precarious one, because the Higgs sector, fundamental as well as elusive as it has turned out to be, has been founded on an unusually compromising mixture of arbitrariness and inelegance. This view may raise objections but, in any case, it accords with the wider one on the need for updated alternatives. If we restrict ourselves, as we will, to the Higgs mechanism, alternatives thereof have apparently been relatively few following its collective formulation in the early 60's. This mechanism, from the initial status of a 'clever artifact' at the time [presumably to be replaced by the then expected proper theoretical development] has since been elevated to a fundamental part of the best theory we have for the EW interaction. If the former is shaken, the latter (and not only) will be in need of even more fundamental reform.

In the present work, the crucial step is to minimally lift the constraint of vanishing torsion¹ in a minimally adopted classical Higgs-less $M^4 \times S^3 \times S^1$ Kaluza-Klein $SU(2) \times U(1)$ gauge-theory vacuum [2]. We will only employ purely geometric quantities (no energy-momentum or other such fields of any kind will be introduced) and we will proceed by fundamental considerations in Physics, as guided by general principles in Riemannian and group-theoretic arguments from holonomy in Riemann-Cartan differential geometry [3],[4].

2 Basic setting and an overview

With some sacrifice of brevity for clarity, we will now introduce notation for the already mentioned 8-dimensional Kaluza-Klein vacuum

$$M^8 = M^4 \times S^3 \times S^1, \quad (2.1)$$

with holonomic coordinates, say, x^μ in M^4 and $y^{\hat{m}}$ in $S^3 \times S^1$. As we will mostly concentrate on the latter, it is advantageously equivalent to employ, as we will, non-holonomic coordinates, implicitly defined by an invariant basis of 1-forms $\ell^{\hat{a}}$. We first note that, strictly speaking, the definition (2.1) holds globally only for the ground-state M^8 direct-product

¹The notion of torsion is here being utilized with its purely geometric interpretation, namely as introduced by Cartan and exemplified by, e.g., Trautman [4], *not necessarily* in the context of the so-called metric-affine or Einstein-Cartan theories of gravity.

manifold. For the menagerie of M^8 manifolds to which one can extend beyond the ground-state M^8 , the definition (2.1) holds only locally, namely only in open neighbourhoods, as defined by, e.g., Cartan frames e^A (namely *local* non-holonomic orthonormal bases of 1-forms), such as the ones we will employ. In any case, as the definition (2.1) is understood to define manifolds (rather than just bare topology), it must be supplemented by whatever pair of metric g_{MN} plus (e.g., Christoffel) connection γ^M_{NP} or, equivalently, by whatever pair of frames e^A plus 1-form connection ω^A_B is being used. This, if needed for clarity, will be shown explicitly as $M^8(e^A, \omega^A_B)$, or $M^8(\omega)$ for short, and understood along with its whatever 8-beins e^A_M and inverse E^M_A . All such M^8 manifolds will automatically have the same components for their metric η_{AB} , by definition identical to their common signature, namely

$$\eta_{AB} = \text{diag}(-1, +1, +1, +1; +1, +1, +1, +1) . \quad (2.2)$$

The range for the world indices A, B, \dots is $A = (\alpha; \hat{a})$, with $\alpha = (0, 1, 2, 3)$, $\hat{a} = (a, 4)$, $a = (1, 2, 3)$, and the same for coordinate (usually holonomic) indices M, N, \dots with $M = (\mu; \hat{m})$, etc. $SU(2) \times U(1)$ group indices I, J, \dots have the same range as indices in the $S^3 \times S^1$ manifold, namely $I = (i, 4)$, $i = (1, 2, 3)$. The duality between e^A and the E_B basis for tangent-vectors in M^8 relates (2.2) to the conventional *holonomic-coordinate* metric g_{MN} as

$$e^A(E_B) = e^A_M E^M_B = \delta^A_B : \quad \eta_{AB} = E^M_A E^N_B g_{MN} \iff g_{MN} = e^A_M e^B_N \eta_{AB} . \quad (2.3)$$

In the case of the ground-state manifold $M^8(e^A, \gamma^A_B)$, or $M^8(0)$ for short, the latter reduces to its familiar block-diagonal expression as

$$e^A_M = \begin{pmatrix} e^\alpha_\mu \\ e^{\hat{a}}_{\hat{m}} \end{pmatrix} \iff g_{MN} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & \hat{g}_{\hat{m}\hat{n}} \end{pmatrix} . \quad (2.4)$$

The set of Christoffel connection 1-forms γ^A_B needs no special mention as it is, of course, always uniquely determined by the metric or the frames. When we lift the zero-torsion constraint (but keep the wider zero-metricity constraint), the general connection is

$$\omega^A_B = \gamma^A_B + K^A_B , \quad (2.5)$$

where the γ^A_B set of 1-forms (which is not a tensor as it remains a Christoffel connection) is antisymmetric in A, B if expressed in the Cartan frames we employ. The contorsion K^A_B is the standard antisymmetric in A, B tensor-valued 1-form, with components related algebraically to those of torsion. Cartan's structure equations

$$\mathcal{T}^A : = \mathcal{D}e^A := de^A + \omega^A_B \wedge e^B = De^A + K^A_B e^B = K^A_B e^B = \frac{1}{2} \mathcal{T}^A_{EP} e^E \wedge e^P , \quad (2.6)$$

$$\mathcal{R}^A_B : = d\omega^A_B + \omega^A_E \wedge \omega^E_B = \frac{1}{2} \mathcal{R}^A_{BEP} e^E \wedge e^P , \quad (2.7)$$

which also involve the covariant exterior derivatives \mathcal{D} and D w.r.t. ω^A_B and γ^A_B with $De^A \equiv 0$, define the fundamental pair of the torsion \mathcal{T}^A and the Riemann-curvature \mathcal{R}^A_B 2-forms. The trace of the former is identical (up to sign) to that of the contorsion K^A_B ; the traces of the Riemann, through the Ricci tensor, produce the curvature scalar R . This allows us to write down the full content of the Hilbert-Einstein action as

$$\mathcal{I}_{H-E} \sim \int_{M^8} \mathcal{R}^A_B \wedge \varepsilon^B_A = \int_{M^8} R \varepsilon, \quad (2.8)$$

with the standard definitions for the M^8 6-form density and volume elements

$$\varepsilon_{A_1 A_2} := \frac{1}{(8-2)!} \epsilon_{A_1 A_2 \dots A_8} e^{A_3} \wedge \dots \wedge e^{A_8}, \quad \varepsilon := \frac{1}{8!} \epsilon_{A_1 A_2 \dots A_8} e^{A_1} \wedge \dots \wedge e^{A_8}. \quad (2.9)$$

We will now proceed with a preliminary and informal overview of our results. All geometric and symmetry aspects (to be properly examined in section 4) are involved in three basic manifolds, related as they evolve from first to last as

$$M^8(0) := M^8(e^A, \gamma^A_B) \xrightarrow{[\xi \mathcal{A}]} M^8(\gamma) := M^8(e^A, \gamma^A_B) \xrightarrow{\text{torsion}} M^8(\omega) := M^8(e^A, \omega^A_B). \quad (2.10)$$

In standard non-abelian Kaluza-Klein theory, the first step in (2.10) is realized with the $SU(2) \times U(1)$ gauge potentials \mathcal{A} , along with the $S^3 \times S^1$ Killing vectors ξ_I , introduced in products as off-diagonal elements in the metric (2.4). We'll do the same thing here with the employment of the scaleless (1,1)-rank 'diagonal' tensor $[\xi \cdot \mathcal{A}]$ with components

$$[\xi \cdot \mathcal{A}]_{\beta}^{\hat{a}} := \xi_i^{\hat{a}} \mathcal{A}_{\beta}^i \sin \theta + \xi_4^{\hat{a}} \mathcal{A}_{\beta}^4 \cos \theta, \quad (2.11)$$

to be formally introduced later on. As it happens, in the first step in (2.10), $[\xi \cdot \mathcal{A}]$ *tilts* (occasionally better visualized as 'shakes-up') the frames with point-depended translations and rotations in M^8 , but under the zero-torsion constraint for the connection; with the second step in (2.10), the frames remain the same but the constraint is lifted and torsion emerges, of course in the context of Riemann-Cartan geometry.

Turning from geometric to symmetry aspects, of particular interest are the S^3 sections (slicings) of the $S^3 \times S^1$ torus in (2.1). In the initial ground-state manifold, this S^3 is a *round* one.² With the first step in (2.10), the round S^3 loses 'half' of its symmetry, now reducing to a *squashed* S^3 , which is only homogeneous, with only its left-invariant 1-forms ℓ^a surviving. With the second step in (2.10), as the zero-torsion constraint is lifted, this squashed S^3 loses all of the remaining symmetry, practically stripped-down to almost bare topology, in its terminal reduction to a *deformed* S^3 . However, any sense of degeneration would be false

²Round, aka maximally symmetric, aka homogeneous and isotropic, aka invariant under the translations and rotations of its full isometry group of motions, aka invariant under the $SU(2) \times SU(2)$ left and right action of $SU(2)$ on its group manifold S^3 .

at this point, because what has actually taken place is a rather miraculous phenomenon of *generation*, namely that of mass from the vacuum; the loss of the last symmetry is directly responsible for the simultaneous emergence of the $SU(2) \times U(1)$ gauge-boson masses, thus created from and as part of pure geometry.

3 Relating to standard K-K $SU(2) \times U(1)$ gauge theory

We begin by first relating our preceding overview to more rigorous notation. The left-invariant non-holonomic set of 1-forms ℓ^a on S^3 can be supplemented with an extra 1-form ℓ^4 for $U(1)$, hence with $d\ell^4 = 0$, thus enlarged to a left-invariant basis $\ell^{\hat{a}}$ on $S^3 \times S^1$, with dual $L_{\hat{a}}$. Their non-vanishing Maurer-Cartan equations and commutation relations are

$$\ell^{\hat{a}}(L_{\hat{b}}) = \delta_{\hat{b}}^{\hat{a}} \implies d\ell^a = -\frac{1}{2}\epsilon_{bc}^a \ell^b \wedge \ell^c \iff [L_b, L_c] = \epsilon_{bc}^a L_a . \quad (3.1)$$

Associated with the group of motions (isometries) on $S^3 \times S^1$, there are four Killing vectors ξ_I which *survive* when the round S^3 in $M^8(0)$ reduces to a squashed one in $M^8(\gamma)$, with components $\xi_I^{\hat{a}}$ which are identical in any one of the $M^8(0)$, $M^8(\gamma)$ or $M^8(\omega)$ manifolds (a property we will shortly expand on in this section). Their non-vanishing commutation relations are

$$\mathcal{L}_{\xi_j} \xi_k := [\xi_j, \xi_k] = \epsilon_{jk}^i \xi_i , \quad (3.2)$$

where \mathcal{L}_{ξ_I} is the Lie derivative for each Killing vector ξ_I . Their orthogonality and lengths are fixed by the *slicing angle* θ , with values in $(0, \pi/2)$, as³

$$\xi_I^{\hat{a}} \xi_J^{\hat{b}} \delta_{\hat{a}\hat{b}} := \left(\frac{L_o}{\sin \theta} \right)^2 \delta_{ij} \delta_I^i \delta_J^j + \left(\frac{L_o}{\cos \theta} \right)^2 \delta_{44} \delta_I^4 \delta_J^4 . \quad (3.3)$$

The scale L_o of the components is imposed by the frames and the lengths $L_o/\sin \theta$ and $L_o/\cos \theta$ are proportional to the radii of S^3 and S^1 in the particular slicing of the $S^3 \times S^1$ torus, as fixed by θ . The ξ_I provide a basis for tangent vectors in $S^3 \times S^1$, just like the earlier introduced $L_{\hat{a}}$. However, while the $L_{\hat{a}}$ are *ab initio* left-invariant, a fact equivalently expressed as⁴

$$\mathcal{L}_{\xi_I} \ell^{\hat{a}} = 0, \quad \mathcal{L}_{\xi_I} L_{\hat{a}} := [\xi_I, L_{\hat{a}}] = 0 , \quad (3.4)$$

³The slicing angle θ should be carefully distinguished from Weinberg's *mixing angle* θ_W . Although fundamentally distinct, θ and θ_W will relate to one-another in a subtle way, apparently nebulous in the literature, as it will emerge in the sequel.

⁴Since the frames $e^{\hat{a}}$ will be constructed from the invariant basis $\ell^{\hat{a}}$, either of the two equations in (3.4) expresses precisely the content of the related Killing equations.

the ξ_I cannot possibly form a left-invariant basis, in view of (3.2). Therefore, under ordinary circumstances, the ξ_I would be an odd and cumbersome (albeit fully legitimate) basis to employ in left-invariant environments, such as those involving the round or even the squashed S^3 's. This observation will be useful to us later on.

We now proceed with the formal definition of the first step in (2.10), which starts with the frames $e^{\overset{o}{A}}$ and dual $\overset{o}{E}_{\hat{b}}$ in $M^8(0)$, introduced as columns and lines, with

$$\begin{aligned} e^{\overset{o}{A}} : \quad & [e^{\overset{o}{\alpha}} = e^{\overset{o}{\alpha}}_{\mu} dx^{\mu} ; \quad e^{\overset{o}{\hat{a}}} = L_o(l_3 \delta^{\hat{a}}_a \ell^a + l_1 \delta^{\hat{a}}_4 \ell^4)], \\ \overset{o}{E}_B : \quad & (\overset{o}{E}_{\beta} = \overset{o}{E}_{\beta}^{\mu} \partial_{\mu} ; \quad \overset{o}{E}_{\hat{b}} = L_o^{-1}(l_3^{-1} \delta^{\hat{b}}_b L_b + l_1^{-1} \delta^{\hat{b}}_4 L_4)) , \end{aligned} \quad (3.5)$$

where the scaleless parameters l_3, l_1 fix the radii of S^3 and S^1 in L_o -length units. With the first step in (2.10), the above frames evolve to

$$e^{\overset{o}{A}} \rightarrow e^A := e^{\overset{o}{A}} + g[\xi \cdot \mathcal{A}]^{\hat{a}} \delta^A_{\hat{a}} \iff \overset{o}{E}_B \rightarrow E_B = \overset{o}{E}_B - g[\xi \cdot \mathcal{A}]_{\beta} \delta^{\beta}_B , \quad (3.6)$$

with E_B following by duality and with g a scaleless coupling parameter.⁵ We have already mentioned $[\xi \cdot \mathcal{A}]$, which is a mixed (1,1)-rank tensor with components

$$[\xi \cdot \mathcal{A}]^A_B := [\xi \cdot \mathcal{A}]^{\hat{a}}_{\beta} \delta^A_{\hat{a}} \delta^{\beta}_B . \quad (3.7)$$

These can be explicitly specified by the (1,0)-valued 1-form and (0,1)-valued tangent vector in (3.6), defined and calculated as

$$\begin{aligned} [\xi \cdot \mathcal{A}]^{\hat{a}} : &= \xi^{\hat{a}}_i \mathcal{A}^i \sin \theta + \xi^{\hat{a}}_4 \mathcal{A}^4 \cos \theta, \\ [\xi \cdot \mathcal{A}]_{\beta} &= \xi_i \mathcal{A}^i_{\beta} \sin \theta + \xi_4 \mathcal{A}^4_{\beta} \cos \theta . \end{aligned} \quad (3.8)$$

We observe that, by moving $g[\xi \cdot \mathcal{A}]$ on the other side of each equation in (3.6), the latter can be made to define $e^{\overset{o}{A}}$ in terms of e^A and $[\xi \cdot \mathcal{A}]$; the question then arises as to which frame are the (3.7) components expressed in. Actually, and this is part of the elegance of the K-K scheme, there is no obscurity or inconsistency involved. The reason is that $[\xi \cdot \mathcal{A}]$ belongs to a class of geometric quantities that have the *same* components in any one of the $M^8(0)$, $M^8(\gamma)$ or $M^8(\omega)$ manifold frames, just like the already-mentioned $\xi^{\hat{b}}_I$. The result is due to two frame and vierbein invariances in (3.6), as stated below, which give rise to that special class, with several examples pointed-to by the long arrow

$$e^{\overset{o}{\alpha}} \rightarrow e^{\alpha} = e^{\overset{o}{\alpha}}, \quad \overset{o}{E}_{\hat{a}} \rightarrow E_{\hat{a}} = \overset{o}{E}_{\hat{a}} \implies e^{\alpha}_{\mu}, \quad E^{\hat{m}}_{\hat{b}}, \quad L^{\hat{m}}_{\hat{b}}, \quad \xi^{\hat{b}}_I, \quad \mathcal{A}^I_{\beta}, \quad [\xi \cdot \mathcal{A}]^{\hat{a}}_{\beta}, \quad \varepsilon_{A_1 \dots A_8}, \quad \partial_y. \quad (3.9)$$

⁵We recall that the basic scales, like L_o (to be thought-of as Planck scale), are carried by the frames. As a rule, all other geometric quantities employed must have derivable scale or be scaleless as, e.g., with all entries in (2.5) and (3.4). In the scaleless coupling parameter $g = \sqrt{2}\kappa/L_o$, the denominator will de-scale ξ^{α}_I , circumstantially scaled by L_o in (3.8), and then $\kappa^2 = 8\pi G_N$ (to be thought-of as the gravitational coupling) will provide the missing scale. Variant reasoning will later reveal the EW-scale parameter L_1 .

Examples of typical behavior or counter-examples to the above involve differing components, e.g., unlike ∂_y in (3.9) and contrasted to it, we have

$$\overset{o}{E}_\beta \rightarrow E_\beta = \overset{o}{E}_\beta - [\xi \cdot \mathcal{A}]_\beta \implies \partial_\mu \rightarrow \partial_\mu - g(\xi_i \mathcal{A}_\mu^i \sin \theta + \xi_4 \mathcal{A}_\mu^4 \cos \theta) , \quad (3.10)$$

expressed in holonomic coordinates too, with $\partial_\mu := \partial/\partial x^\mu$. The latter supplies us with a generalized minimal-coupling prescription, here obtained as a rigorous result from (3.6). On the same grounds, and in view of the $\overset{o}{\varepsilon} = \varepsilon$ result in (3.9) for all M^8 volume elements, the Hilbert-Einstein action (2.8) will also evolve along the (2.10) sequence, as depicted in

$$\mathcal{I}_{HE}(0) \sim \int_{M^8(0)} R(0) \varepsilon \xrightarrow{[\xi \cdot \mathcal{A}]} \mathcal{I}_{HE}(\gamma) \sim \int_{M^8(\gamma)} R(\gamma) \varepsilon \xrightarrow{\text{torsion}} \mathcal{I}_{HE}(\omega) \sim \int_{M^8(\omega)} R(\omega) \varepsilon, \quad (3.11)$$

where $R(0)$, $R(\gamma)$ and $R(\omega)$ are the curvature scalars in the respective manifolds.

4 Torsion as prescribed by geometry and symmetry

Here we will essentially review more or less known results in standard Kaluza-klein $SU(2) \times U(1)$ gauge theory, and recall some results from holonomy (as they relate to our present needs and discussed in the last section); the aim is to explicitly deduce our minimally adequate torsion from fundamental geometric and symmetry aspects, before we proceed with our main results in the next section. The Lagrangians \mathcal{L}_{HE} , which can be read-off from the actions (3.11) as proportional to the corresponding curvature scalars R , are directly calculable from the geometric definitions (2.5-2.8). With the usual adjustments,⁶ we obtain the well-known result for $M^8(\gamma)$ as

$$\mathcal{L}_{HE}(\gamma) = \mathcal{L}_{HE}(0) - \frac{1}{2} \kappa^2 \mathcal{F}^2 . \quad (4.1)$$

The calculation may begin with the defining setup in the previous section for the basic preliminary result

$$de^{\hat{a}} = g(\sin \theta \xi_i^{\hat{a}} \mathcal{F}^i + \cos \theta \xi_4^{\hat{a}} \mathcal{F}^4) - \frac{1}{2l_3 L_o} \delta_a^{\hat{a}} \epsilon_{bp}^a e^b \wedge e^p . \quad (4.2)$$

The field strengths therein have been introduced as

$$\mathcal{F}^I := d\mathcal{A}^I + \frac{1}{2} g \sin \theta \delta_i^I \epsilon_{jk}^i \mathcal{A}^j \wedge \mathcal{A}^k , \quad (4.3)$$

⁶With hardly any non-trivial exception, the curvature scalars R contain surface terms *omitted* in the corresponding Lagrangians. However, the latter differ from R also with *added* terms, notably effective cosmological constants coming from reduction to 4 spacetime dimensions. Both adjustment are here understood as present in the ground state Lagrangian $\mathcal{L}_{HE}(0)$.

in an obviously left-invariant environment, leading to (4.1) with

$$\mathcal{L}_{\xi_I} \mathcal{F}^J = 0, \quad \mathcal{L}_{\xi_I} \mathcal{L}_{HE}(\gamma) = 0. \quad (4.4)$$

To repeat the calculation for $\mathcal{L}_{HE}(\omega)$, we observe that the connection ω in (2.5) for $M^8(\omega)$ involves the additional contribution from the contorsion tensor K^A_B . Without any restriction for the latter, the result of this calculation is

$$\mathcal{L}_{HE}(\omega) = \mathcal{L}_{HE}(0) - \frac{1}{2} \kappa^2 \mathcal{F}^2 + K^{APB} K_{BPA} - (\text{tr} K)^2. \quad (4.5)$$

We still have a left-invariant \mathcal{F}^I , with the same definition (4.3), and with the first equation in (4.4) still valid. However, as we expect, this cannot be the case for the contorsion tensor K^A_B , hence it will no-longer be the case for the Lagrangian $\mathcal{L}_{HE}(\omega)$ either.

The result from holonomy (to be briefly reviewed in the last section, as mentioned) is that the torsion tensor \mathcal{T}^A , as it emerges in the present context, will have to be proportional to the gauge potentials \mathcal{A}^I ; we thus need to clarify whether the latter should enter the specification of \mathcal{T}^A as $[\xi \cdot \mathcal{A}]$ or as $\xi_I \mathcal{A}^I$. The choice is already rooted in the contemplation following (3.4), as to why a non-invariant (albeit legitimate) basis ξ_I is employed in the left-invariant environment of $M^8(\gamma)$. In fact, to just arrive at (4.1) or even (4.5), we could have entirely dispensed with the slicing-angle θ and the Killing vectors, altogether. This is already quite obvious from (4.4) and the trivially redundant presence of θ in (4.3). It can also be explicitly demonstrated with the employment of the left-invariant basis, easily effected with the symmetric alternative $L_o \delta_I^{\hat{a}}$ replacing $\xi_I^{\hat{a}}$ in $[\xi \cdot \mathcal{A}]$, formally with

$$[\xi \cdot \mathcal{A}]^{\hat{a}} \rightarrow [\text{I} \cdot \mathcal{A}]^{\hat{a}} := L_o \delta_I^{\hat{a}} \mathcal{A}^I \quad (4.6)$$

in (3.6). The calculation, now significantly simplified in the explicitly symmetric environment, leads (modulo trivial re-definitions) to precisely the same Lagrangians (4.1) and (4.5). The compatibility of the use of two fundamentally different frames, namely the non-invariant ξ_I and the left-invariant $L_{\hat{a}}$ (or, equivalently, the $E_{\hat{a}}$ in (4.1)) is achieved as follows: The particular choice of dependence of $[\xi \cdot \mathcal{A}]$ on θ in (3.8) is made in conjunction with the *ad hoc* choice of the radii $L_o/\sin \theta$ and $L_o/\cos \theta$ in the (3.3) slicing, with the intention to precisely cancel out the θ -dependence. This renders any θ -slicing of the $S^3 \times S^1$ torus as good as any other, therefore it *explicitly* re-establishes left-invariance, which was actually never lost. However, all this indeed redundant involvement of θ -slicing is not only *not* useless, it is, in fact, crucial and irreplaceable, as we will see in a moment. We now let a mixing-angle θ_W introduce through K^A_B a randomly occurring symmetry-breaking direction, say $\Xi^{\hat{a}}$, tangent to the $S^3 \times S^1$ torus in $M^8(\omega)$. With proper normalization to unit length and without loss of generality, we follow the conventional choice in context, with

$$\Xi^{\hat{a}} := \frac{1}{\sqrt{2}L_o} (\xi_3^{\hat{a}} \sin \theta_W + \xi_4^{\hat{a}} \cos \theta_W), \quad (4.7)$$

inducing, correspondingly, the conventional mixing of $SU(2)\times U(1)$ gauge potentials. The S^3 sections, now identified by the $\theta = \theta_W$ slicings,⁷ are precisely the deformed S^3 's at every point in $M^8(\omega)$, as already described in section 2.

We now have two results at hand, first to opt for $\xi_I \mathcal{A}^I$ rather than $[\xi, \mathcal{A}]$ (in view of the circumstantially *ad hoc* symmetry-restoring rôle of the latter), and second the symmetry-breaking $\Xi^{\hat{a}}$ vector in (4.7), as they complement the mentioned holonomy-group requirement of proportionality of the torsion tensor \mathcal{T}^A to \mathcal{A}^I .⁸ The result (essentially unique by minimality) for our torsion dictates *only* \mathcal{T}^α components, properly scaled as⁹

$$\mathcal{T}^\alpha \sim \frac{g}{L_1} \delta^{\alpha\beta} \Xi^{[\hat{a}} \xi_I^{\hat{b}]} \mathcal{A}_\beta^I . \quad (4.8)$$

In the next section we turn to our main results, presented in detail sufficient for their reproduction.

5 $SU(2)\times U(1)$ gauge-boson masses from torsion

Conforming to standard notation in anticipation of the presence of a gauge-boson mass term (with the correct sign!) in the Lagrangian (4.5), we may re-write it as

$$\mathcal{L}_{HE}(\omega) = \mathcal{L}_{HE}(0) - \frac{\kappa^2}{2} \mathcal{F}^2 - \kappa^2 M_{IJ} \mathcal{A}_\alpha^I \mathcal{A}_\beta^J \delta^{\alpha\beta} , \quad (5.1)$$

wherefrom, observing that the torsion in (4.8) is traceless (therefore, so is the contorsion it produces) we read off (4.5)

$$\kappa^2 M_{IJ} \mathcal{A}_\alpha^I \mathcal{A}_\beta^J \delta^{\alpha\beta} = -K^{APB} K_{BPA} . \quad (5.2)$$

In the context of our definitions, the components of the torsion and contorsion tensors are interrelated as

$$K_{ABP} = -\frac{1}{2} (T_{ABP} + T_{BPA} - T_{PAB}) \quad \Longleftrightarrow \quad T_{ABP} = -K_{ABP} + K_{APB} , \quad (5.3)$$

⁷Here, the subtle interrelation between θ and θ_W we referred-to earlier is rather clear. By being *a posteriori* set equal to θ_W , θ itself overrides its until-then redundant presence, while θ_W acquires an additional, now ‘slicing’ property, which it couldn’t have otherwise. However, with $\theta = \theta_W$, the until-then valid ‘any θ slicing is as good as any other’ is obviously lost, subject to the same symmetry breaking as induced by (4.7). Of course, in the absence of symmetry breaking, θ is trivially redundant and θ_W is trivially irrelevant.

⁸Viewed as a vector-valued form in $M^8(\omega)$, \mathcal{T}^A must be proportional to the gauge potentials not just numerically but essentially to their vectorial directions $\delta^{\alpha\beta} \mathcal{A}_\beta^I$, tangent to the M^4 subspaces of $M^8(\omega)$.

⁹The components of \mathcal{T}^α carry a scale from whatever frames employed, here L_o from $e^{\hat{a}}$, exactly as the dimensionless \mathcal{F} does from e^α , but, unlike the latter, the *dimensionful* \mathcal{T}^α requires an extra frame-independent length-scale, here supplied by L_1 . A more rigorous version of this claim follows from Cartan’s first structure equation in (2.6), which quantifies the enlargement of geometry with torsion.

so our basic result in (4.8) supplies for our torsion and contorsion tensors their only non-vanishing components as

$$-K^{\alpha\hat{b}\hat{p}} = +K^{\hat{b}\alpha\hat{p}} = +K^{\hat{b}\hat{p}\alpha} = +\frac{1}{2}T^{\alpha\hat{b}\hat{p}} = \frac{g}{L_1}\delta^{\alpha\beta}\Xi^{[\hat{b}\xi\hat{p}]}_{\xi I}\mathcal{A}^I_{\beta}. \quad (5.4)$$

The straightforward substitution of (5.4) in (5.2) quantifies the mass matrix as

$$M_{IJ} = (L_o L_1)^{-2} [(\Xi)^2 \xi_I^{\hat{p}} \xi_J^{\hat{q}} \delta_{\hat{p}\hat{q}} - (\Xi^{\hat{p}} \xi_I^{\hat{q}} \delta_{\hat{p}\hat{q}}) (\Xi^{\hat{r}} \xi_J^{\hat{s}} \delta_{\hat{r}\hat{s}})] , \quad (5.5)$$

where $(\Xi)^2 = 1$ and

$$\Xi^{\hat{p}} \xi_I^{\hat{q}} \delta_{\hat{p}\hat{q}} = \frac{L_o}{\sqrt{2}} \left(\frac{\sin \theta_W}{\sin^2 \theta} \delta_{3I} + \frac{\cos \theta_W}{\cos^2 \theta} \delta_{4I} \right) . \quad (5.6)$$

After setting $\theta = \theta_W$ (as we will do from now on), we introduce for brevity

$$(\Xi \cdot \xi)_I := \Xi^{\hat{p}} \xi_I^{\hat{q}} \delta_{\hat{p}\hat{q}} ([\theta = \theta_W]) = \frac{L_o}{\sqrt{2}} \left(\frac{1}{\sin \theta_W} \delta_{3I} + \frac{1}{\cos \theta_W} \delta_{4I} \right) , \quad (5.7)$$

so, using (3.3), (4.7) and (5.7), we may express (5.5) as

$$\begin{aligned} M_{IJ} &= (L_o L_1)^{-2} [\xi_I^{\hat{p}} \xi_J^{\hat{q}} \delta_{\hat{p}\hat{q}} - (\Xi \cdot \xi)_I (\Xi \cdot \xi)_J] \\ &= (L_1 \sin \theta_W)^{-2} \left[\delta_{ij} \delta_I^i \delta_J^j - \frac{1}{2} (\delta_I^3 \delta_J^3 + \tan^2 \theta_W \delta_I^4 \delta_J^4 + \tan \theta_W (\delta_I^3 \delta_J^4 + \delta_I^4 \delta_J^3)) \right] , \end{aligned} \quad (5.8)$$

or, in the more conventional matrix notation,

$$M_{IJ} = (L_1 \sin \theta_W)^{-2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \tan \theta_W \\ 0 & 0 & -\frac{1}{2} \tan \theta_W & \frac{1}{2} \tan^2 \theta_W \end{pmatrix} . \quad (5.9)$$

Either of

$$\Delta^I_J = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\cos \theta_W & \sin \theta_W \\ 0 & 0 & +\sin \theta_W & \cos \theta_W \end{pmatrix} \text{ or } \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -i/\sqrt{2} & i/\sqrt{2} & 0 & 0 \\ 0 & 0 & -\cos \theta_W & \sin \theta_W \\ 0 & 0 & +\sin \theta_W & \cos \theta_W \end{pmatrix} \quad (5.10)$$

diagonalizes M_{IJ} to its eigenvalues as

$$M^I_J = (L_1 \sin \theta_W)^{-2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2}(\cos \theta_W)^{-2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} m_W^2 & 0 & 0 & 0 \\ 0 & m_W^2 & 0 & 0 \\ 0 & 0 & \frac{1}{2}m_Z^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} . \quad (5.11)$$

The acquired gauge-boson masses (with their spectrum also interpretable as angular ‘spring-constants’ of the anisotropic vacuum, as we will see), are fixed as

$$m_W = (L_1 \sin \theta_W)^{-1}, \quad m_Z = (L_1 \sin \theta_W \cos \theta_W)^{-1}, \quad (5.12)$$

with the ρ parameter, which is defined as $\rho := m_W^2 / (m_Z \cos \theta_W)^2$, post-dicted directly from (5.12) as $\rho = 1$. In terms of the usual expressions for the physical gauge bosons, actually read off the columns of the second diagonalizing matrix in (5.10), we have

$$W^\pm = \frac{1}{\sqrt{2}}(\mathcal{A}^1 \mp i\mathcal{A}^2), \quad Z = -\cos \theta_W \mathcal{A}^3 + \sin \theta_W \mathcal{A}^4, \quad B = \sin \theta_W \mathcal{A}^3 + \cos \theta_W \mathcal{A}^4, \quad (5.13)$$

so (5.2) takes on the standard expression for the mass term as

$$\kappa^2 M_{IJ} \mathcal{A}_\alpha^I \mathcal{A}_\beta^J \delta^{\alpha\beta} = \kappa^2 \left(m_W^2 W^+ W^- + \frac{1}{2} m_Z^2 Z^2 \right). \quad (5.14)$$

6 Discussion

We may firstly re-focus on certain points in our preliminary overview in section 2. The tilt (3.6) of the frames, on occasion viewable as a ‘shake-up’, is *not* a perturbation because it can be as steep or violent as it may, restricted only by the general requirement of at least C^2 differentiability. In addition to its obviously translational character, the same tilt involves rotations too. The latter emerge as a consequence of the loss of the *global* hypersurface-forming property of the frames¹⁰ [5], already hinted-to in relation to (2.1).

The fundamental pair of torsion \mathcal{T}^A and curvature \mathcal{R}_B^A , as introduced by Cartan’s structure equations in (2.6) and (2.7), is interrelated by the holonomy theorems to, respectively, the translations and rotations of the inhomogeneous group acting in the tangent spaces of whatever manifold they inhabit [4], here the $M^8(\omega)$. The effect of the specific translations and rotations induced by the tilt (3.6) in their local action in $M^8(\omega)$ can be measured (after the general case) with a construction process for local geodesic quadrilaterals-to-be, by their failure to close; and with the parallel transport of a tangent vector around a geodesic quadrilateral, by its failure to return aligned to its initial direction. The non-closure displacement, a *translation* element, is a local measure of non-vanishing torsion \mathcal{T}^A . The non-alignment

¹⁰This can be better visualized in examples of empty rotating spacetimes in $d = 4$, such as the Kerr solution or the Gödel universe, where the locally orthogonal-to-time $dt = 0$ hypersurfaces do not mesh to allow globally consistent simultaneity. This result is due to a tilt of the frames exactly as in (3.6), with the spacetime vorticity proportional to the time derivative of that tilt. In the present context we have the gauge potentials \mathcal{A}^I as tilt, with the field strengths \mathcal{F}^I as *vorticity*.

angle, a *rotation* element, is a local measure of non-vanishing curvature \mathcal{R}_B^A . The first element accumulates linearly, as two tangent vectors are being transported parallelly and under torsion-caused spiralling from an initial vertex of the (failing-to-close) quadrilateral, along each-other's geodesics and gaining in 'potential energy', pretty much as with a winding coil. There is no differentiation involved in this transport so, in our case, the produced element has to be proportional to the \mathcal{A}^T of the tilt which caused it. The torsion is therefore likewise proportional, hence it contributes with the quadratic-in- \mathcal{A}^T mass term ('potential energy') to $\mathcal{L}_{HE}(\omega)$ in (5.1). The second element accumulates from the entire surface of a closed quadrilateral and it is one level of differentiation up, so, in our case, it expectedly involves $(\mathcal{F})^2$, also viewable as a kinetic (*vorticity*)² term contribution to $\mathcal{L}_{HE}(\omega)$. The interplay between translational and angular degrees of freedom in the above semi-qualitative interpretation follows in geometric elegance from their involvement in the rigorous interrelation (by the mentioned theorems on holonomy) between on one side the Noether currents and charges from (tangent-space) invariances of the Poincarè group as *sources* and, on the other, the generated *fields* of torsion and curvature ([4]).

We may now argue that the above *co-existence* of these two terms in $\mathcal{L}_{HE}(\omega)$, with both having been created simultaneously and by common cause (namely the tilt or shake-up (3.6) of the frames, as seen) is *compulsory* in the sense that having one of these terms without the other would mean direct violation of the mentioned basic theorems on holonomy. By the latter, this violation may also be viewed as that of 'mechanical-energy conservation', with none of its kinetic and potential-energy contributions omissible. In the underlying angular spring-oscillator dynamics, the anisotropic spring-constants of the vacuum are identified with the eigenvalues of the mass matrix. A consequence of this result is that nature must skip the middle step in the evolution scheme (2.10), where the massless case (4.1) cannot be realized within the typical energy range of the standard EW interaction. Of course, at sufficiently high energies, as in the LHC experiment, phenomenology related to the existence of the middle step in (2.10) may well be expected. In the conventional treatment the massless case cannot be excluded from the outset. To elevate the piecemeal addition of $[4(4-3)/2]$ graviton plus $(8+7)$ $SU(2) \times U(1)$ massless gauge-boson and scalar]=17 independent states in the symmetric Lagrangian (4.1) to 20 when the gauge bosons acquire mass *there*, the Goldstone's theorem is invoked for the emergence of the 3 transverse states, accounted for by a simultaneous re-arrangement of the independent states of the Higgs fields and the Goldstone bosons [2]. *Here*, in sharp contrast, the correct count of 20 follows directly as $8(8-3)/2$ for the $M^8(\omega)$ manifold while the massless case (4.1) is disallowed at the mentioned energy range, as seen. The eigenvalues in (5.11) reproduce precisely the spectrum of the well-known experimental result, along with the parallel interpretation of the same mass spectrum as 'spring-constants' of the vacuum, as explained.

Sufficiently close or above the very high energy scale of the massless EW interaction, the ground-state manifold approach for the Kaluza-Klein gauge-theory vacuum (and any tilt of the frames therein) would have already been rendered inapplicable. The so-called *cylinder condition* would have to be abandoned for a more primitive state of the $M^8(\omega)$ manifold. Under such considerations, and if we were to restrict ourselves to the gravitational and EW interactions, it is tempting to conjecture or speculate that the $S^3 \times S^1$ sector of the topology in (2.1) might be expected to emerge with even deeper importance, due its *mixmaster* behavior [5]. This refers to Misner's profoundly non-linear dynamics which could turn *turbulent*¹¹ in the presence of sufficiently steep potential walls. Then, through Kasner-like bounces on them, at Planck-scale frequencies, such dynamics may enforce isotropy and homogeneity on the geometry actually or, better, effectively. The latter case may be realized as seen from a sufficiently longer time scale or as averaged at a much-lower energy scale. It may then be possible that what we assume as the cylinder condition, or employ as a static $S^3 \times S^1$ sector in $M^8(0)$ at the much lower EW energy scale of $\mathcal{L}_{HE}(\omega)$ in (5.1), is the *effectively* static presence of an actually turbulent vacuum dynamics at the much deeper Planck scale. Such studies could justify the cylinder condition and illuminate the question of the classical stability of the ground-state manifold, possibly in relation to its quantum mechanical phenomenology.

If encouraged by the present development (and before any dimensional enlargement of the geometry), one might investigate other types of components of the torsion tensor, in the context of holonomy as mentioned, and in relation to the invariants (spin and mass) and representations of the Poincaré group. In the wider area of the present work there have been earlier contributions with torsion-related aspects and potentially observable effects and testable consequences [6]. Those which may be carried over to the present context include certain Aharonov-Bohm type of gravito-EW interferences and gyro-magnetic effects, as well as a lower bound for the violation of the principle of equivalence, expectedly at roughly the order of $L_1^{-1}/L_o^{-1} \sim 1 : 10^{17}$ (existing tests are negative down to $\sim 1 : 10^{12}$). Additional effects may be revealed as, e.g., with the study of couplings and geodesics independently or under (3.10), now understood with $\theta = \theta_W$.

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¹¹We choose the term 'turbulent' to mean chaotic and micro-causality violating vacuum dynamics, hence one with classical variables which would violate Bell's inequalities, just like quantum mechanical ones. The latter character should also prescribe a specially needed handling of causal transforms, e.g., Fourier expansions, which would in principle be meaningless in such environments.

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